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Optimal Solar Sail Interplanetary Missions with Control Constraints

1. Introduction
2. Mathematical model
3. Problem solution
4. Simulation results
5. Concluding remarks

Introduction

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2. The requirement of orienting and continuously controlling a structure whose characteristic dimension is of the order of some tens of meters is a demanding task.
3. One of the primary goals during the mission analysis phase consists in the development of steering laws capable of combining effectiveness with simplicity to implement.
4. As the objective of minimizing the mission time typically conflicts with that of using simple steering laws, it is often necessary to look for compromise solutions.

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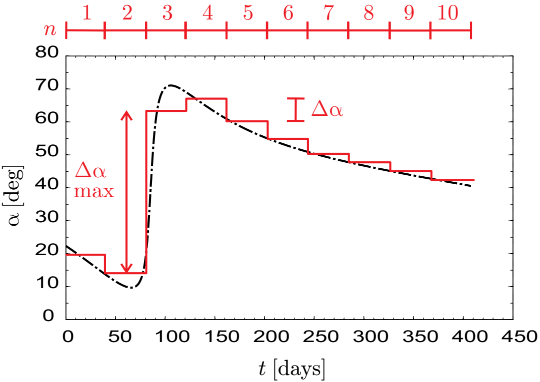
Simplification means **discretization**. Two of the conflicting requirements are the **maximum number of reorientation maneuvers** n, and the maximum value of **reorientation angle** $\Delta\alpha_{\max}$ (ideally this maneuver happens instantaneously).

- $n \rightarrow$ maneuver complexity
- $\Delta\alpha \rightarrow$ actuator effectiveness



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2. **New approach:** The maximum number of different orientation angles allowed by the control law is now an input parameter, while an indirect method is employed to optimally select the control angles in the admissible set and generate the piecewise-constant steering law.
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Force model

Solar radiation pressure

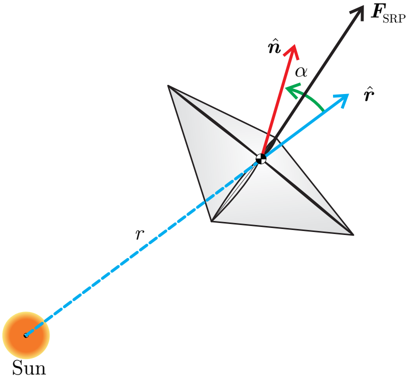
$$P = P_\oplus \left(\frac{r_\oplus}{r}\right)^2, \quad P_\oplus \triangleq 4.563\,\mu\text{N}/\text{m}^2$$

Ideal force model

$$\boldsymbol{F}_{\text{SRP}} = 2\,P\,A\,\cos^2\alpha\,\hat{\boldsymbol{n}}$$

Optical force model

$$\boldsymbol{F}_{\text{SRP}} = 2\,P\,A\,\cos\alpha\left[b_1\,\hat{\boldsymbol{r}} + \right.$$
$$\left. + \left(b_2\,\cos\alpha + b_3\right)\,\hat{\boldsymbol{n}}\right]$$



Definitions

- $A \triangleq$ solar sail area
- $r_\oplus \triangleq$ 1 AU
- $\alpha \triangleq$ sail pitch angle, $\cos\alpha = \hat{\boldsymbol{r}} \cdot \hat{\boldsymbol{n}}$
- $b_i \triangleq$ dimensionless optical force coefficients

Problem setup

- **Assumption:** two-dimensional problem, circle-to-circle transfer

- **State vector**

$$x \triangleq [r \quad \theta \quad u \quad v]^{\mathsf{T}}$$

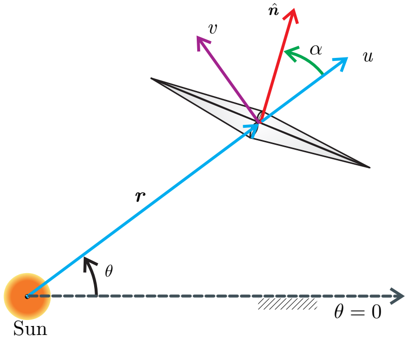
- Heliocentric equations of motion

$$\dot{x} = f_g + f_p$$

- **Problem:** Find the minimum-time steering law $\alpha(t)$

$$J \triangleq -t_f \qquad \Rightarrow \text{maximize}$$

- **Remark:** f_g is independent of α.



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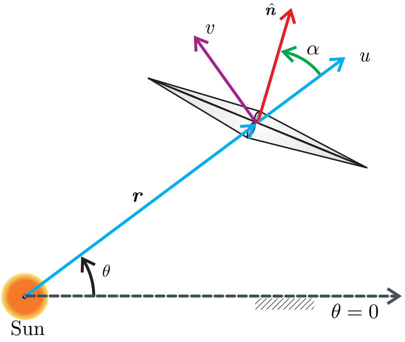

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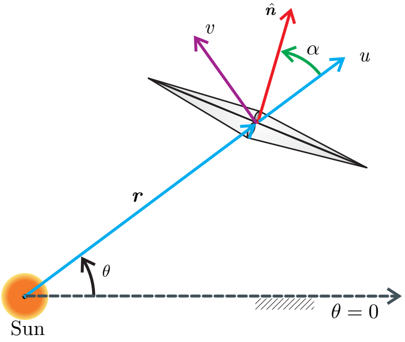

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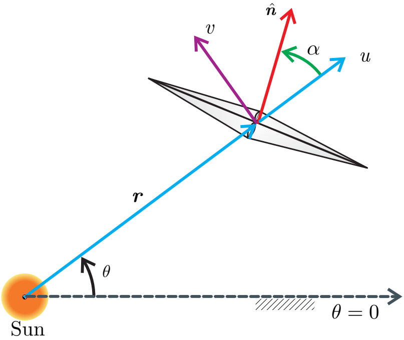

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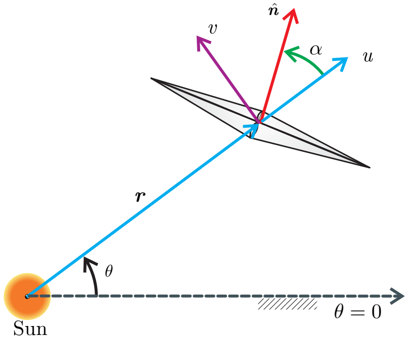

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Two point boundary value problem

1. Introduce the Hamiltonian

$$H \triangleq (\boldsymbol{f}_g + \boldsymbol{f}_p) \cdot \boldsymbol{\lambda}, \qquad \boldsymbol{\lambda} \triangleq [\lambda_r \quad \lambda_\theta \quad \lambda_u \quad \lambda_v]^{\mathsf{T}}$$ **adjoint vector**

2. Combine the equations of motion with the Euler-Lagrange Equations

$$\dot{\boldsymbol{x}} = \boldsymbol{f}_g + \boldsymbol{f}_p$$ **(4 scalar equations)**

$$\dot{\boldsymbol{\lambda}} = -\left(\frac{\partial \boldsymbol{f}_g}{\partial \boldsymbol{x}} + \frac{\partial \boldsymbol{f}_p}{\partial \boldsymbol{x}}\right) \cdot \boldsymbol{\lambda}$$ **(4 scalar equations)**

3. Use the boundary conditions at t_0 and t_f

$$r(t_0) = r_\oplus, \quad \theta(t_0) \equiv u(t_0) = 0, \quad v(t_0) = \sqrt{\mu_\odot / r_\oplus}$$

$$r(t_f) = r_f, \quad u(t_f) \equiv \lambda_\theta(t_f) = 0, \quad v(t_f) = \sqrt{\mu_\odot / r_f}$$

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Optimal steering law

1. Introduce the **primer vector**

$$\boldsymbol{\lambda}_{\mathsf{v}} \triangleq [\lambda_u \quad \lambda_v]^{\mathsf{T}}$$

2. Use the maximum principle to get

$$\alpha = \arg\max_{\alpha \in \mathcal{U}} H_p \qquad \text{with} \qquad H_p \triangleq \boldsymbol{F}_{\text{SRP}} \cdot \boldsymbol{\lambda}_{\mathsf{v}}$$

where $\mathcal{U}$ is the domain of feasible controls.

- **Assumption:** $\mathcal{U}$ is a set of admissible values

$$\mathcal{U} = \{\alpha_1, \alpha_2, \ldots, \alpha_k\}, \qquad k \in \mathbb{N}$$

At a generic time instant the optimal value $\alpha^\star$ for the sail pitch angle is found with a **sorting procedure** in the set of admissible values

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Simulation assumptions

1. The optical force coefficients correspond to a sail with a highly reflective aluminum-coated front side and a highly emissive chromium-coated back side

$$b_1 = 0.0864, \quad b_2 = 0.8272, \quad b_3 = -5.45 \times 10^{-3} \qquad \left(\sum_i b_i < 1\right)$$

2. The final boundary constraints are 1 000 km for the position error and 0.1 m/s for the velocity error
3. The simplest choice of admissible control values is

$$\mathcal{U} = \{-90,\ -\alpha_f,\ 0,\ \alpha_f,\ 90\}\ \text{deg}$$

4. The trajectory complexity can be measured by the number n of sail reorientation maneuvers required by the steering law

✴ **Remark:** n is **different** from the number of arcs characterizing a direct optimization method.

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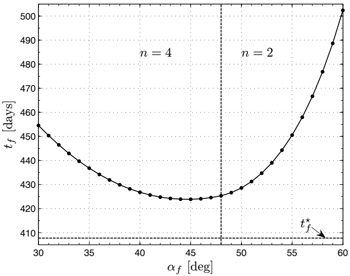
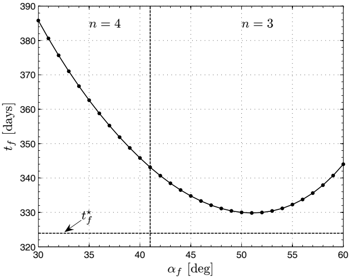
Earth-Mars rendezvous mission

$\mathcal{U} = \{-90,\ -\alpha_f,\ 0,\ \alpha_f,\ 90\}$ deg

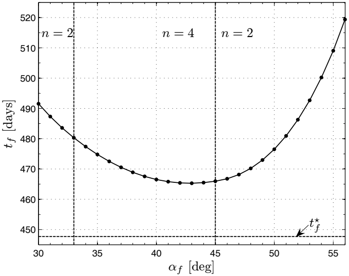
$a_c = 1\ \mathrm{mm/s^2}$
Ideal model

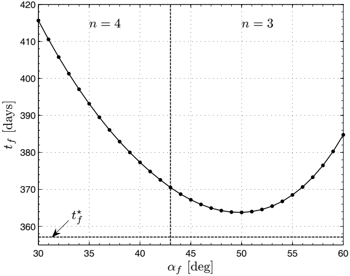




$a_c = 2\ \mathrm{mm/s^2}$
Ideal model

$a_c = 1\ \mathrm{mm/s^2}$
Optical m.





$a_c = 2\ \mathrm{mm/s^2}$
Optical mod.

Earth-Venus rendezvous mission

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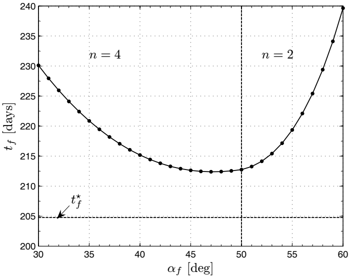

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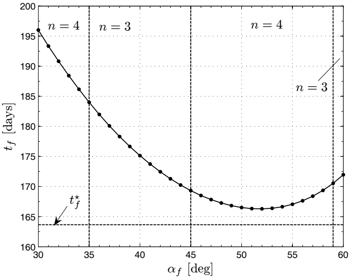

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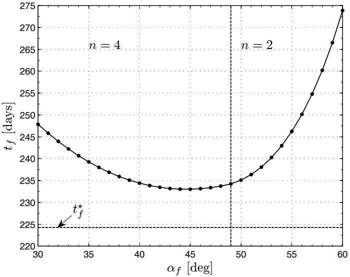

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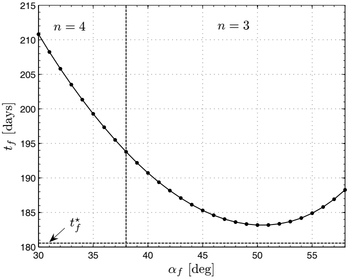


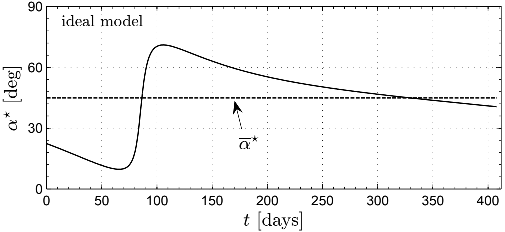
Remarks on the results

1. There exists an optimal value for α_f, that is

$$\alpha_f^\star \triangleq \arg\min_{\alpha_f} \; t_f$$

2. $\alpha_f^\star$ depends both on the value of a_c and on the solar sail force model.
3. It may be shown that $\alpha_f^\star$ is nearly equal to the modulus of the integral mean value $\overline{\alpha}^\star$ of the optimal sail pitch angle

$$\overline{\alpha}^\star \triangleq \frac{s}{t_f^\star} \int_0^{t_f^\star} |\alpha^\star| \, \mathrm{d}t \qquad \text{with} \qquad s \triangleq \operatorname{sign}\left(\int_0^{t_f^\star} \alpha^\star \, \mathrm{d}t \right)$$



Earth-Mars mission
$a_c = 1 \mathrm{mm/s}^2$

Earth-Mars mission with $\alpha_f = \alpha_f^\star$

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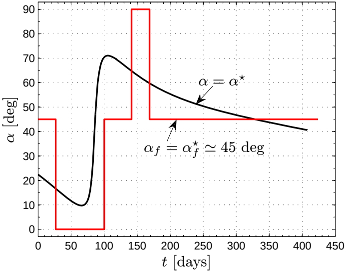
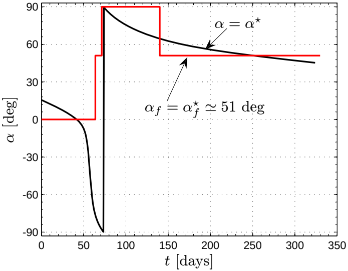
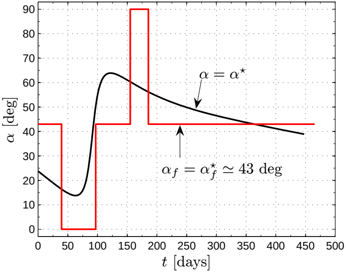
$a_c = 1$ mm/s^2
Ideal model
$(n = 4)$

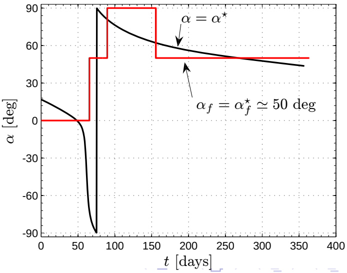




$a_c = 2$ mm/s^2
Ideal model
$(n = 4)$

$a_c = 1$ mm/s^2
Optical m.
$(n = 4)$





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Optical mod.
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Tradeoff performance

- What is the solution sensitivity to the number of admissible values of $\mathcal{U}$?
- The interval $[-90, 90]$ deg is discretized, using a finite number of equispaced admissible values. The angular step size in degrees is

$$\Delta\alpha = \frac{90}{i}, \qquad i \in \mathbb{N}$$

and

$$\mathcal{U} = \{-90 : \Delta\alpha : 90\}, \qquad k = 2\,i + 1$$

- **Example:**

$$\begin{array}{lll} i = 2, & \Delta\alpha = 45\,\text{deg}, & \mathcal{U} = \{-90,\ -45,\ 0,\ 45,\ 90\} \\ i = 3, & \Delta\alpha = 30\,\text{deg}, & \mathcal{U} = \{-90,\ -60,\ -30,\ 0,\ 30,\ 60,\ 90\} \\ i = 4, & \Delta\alpha = 22.5\,\text{deg}, & \mathcal{U} = \{-90,\ -67.5,\ -45,\ -22.5,\ 0,\ \ldots\} \end{array}$$

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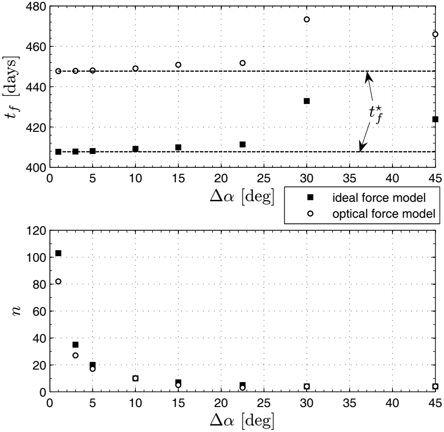
Tradeoff performance for Earth-Mars mission



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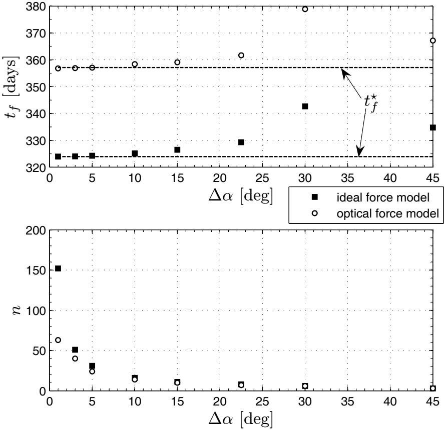

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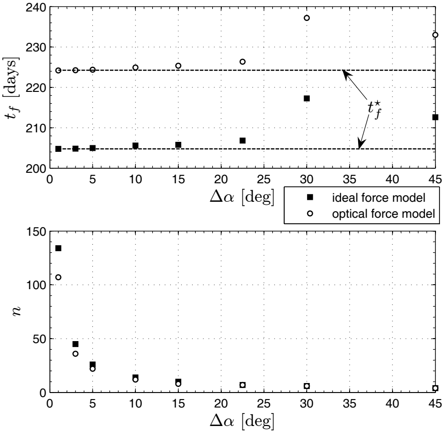
Tradeoff performance for Earth-Venus mission



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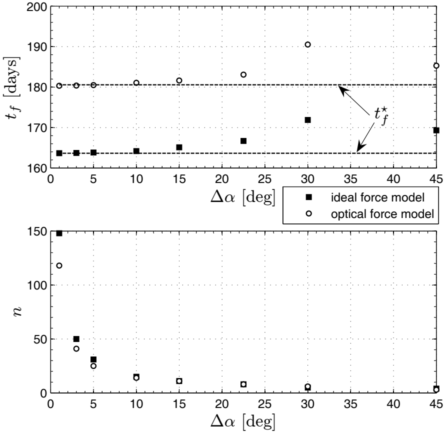

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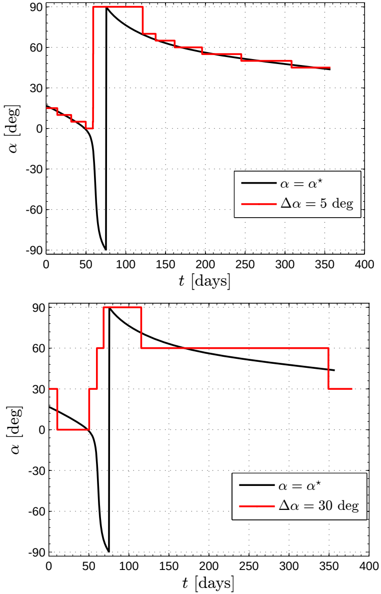
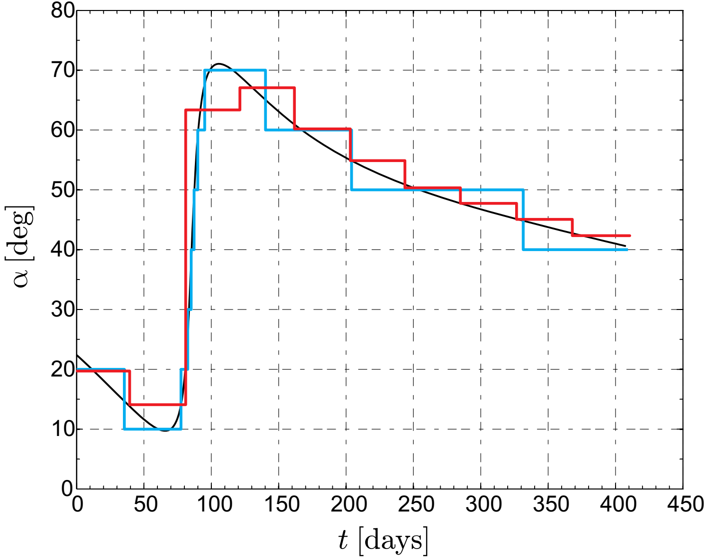
Earth-Mars mission with optical force model and $a_c = 2$ mm/s^2



Comparison with a direct approach (red)

Mission	n	time [days]	$\Delta\alpha$ (max) [deg]	penalty [%]
Earth-Mars (ideal model)	4	423.9 (418.0)	45. (52.)	3.96 (2.40)
	5	411.4	22.5	0.90
	7	409.9	15.	0.55
	10	409.2 (409.0)	10. (49.)	0.36 (0.20)
	20	408.1 (408.8)	5.	0.10 (0.15)
	35	407.9	3.	0.04
	103	407.8	1.	0.015
	∞	407.7	0	0

Direct (red) vs indirect (cyan) performance comparison ($n = 10$)
Earth Mars mission with ideal solar sail



Conclusions

- The minimum time problem of a solar sail using piecewise-constant steering laws can be solved efficiently using an indirect approach.
- When a unique value of sail orientation must be maintained for the whole mission, the optimal choice consists in choosing the integral mean value of the pitch angle calculated with respect to the continuous steering law.
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- For both Earth-Mars and Earth-Venus transfers, with a few values of admissible pitch angles the optimal mission time is close to that found with a continuous steering law.
- Assuming $\Delta\alpha = 15\,\text{deg}$, the number of reorientation maneuvers is $10 \leq n \leq 15$ for all of the simulations.
- A substantial reduction of the reorientation maneuver complexity is possible with results competitive in performance with the optimal variable direction program.
- Unlike a direct approach, the proposed technique allows one to avoid an a-priori discretization of the mission into sub arcs.
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